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Conference Paper

Quantization and Assessment of the Gravitational Waves

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ABSTRACT

General Relativity describes the movement of bodies in strong gravitational fields with the geometrical structure of the dynamical space-time continuum. Accelerating objects produce changes in the curvature which propagate outwards at the speed of light in a wave-like manner which transport energy as gravitational radiation and this phenomenon are known as gravitational waves.

Keywords: Gravitation, Quadruple waves, Minkowskian metric, and Energy momentum tensor.

INTRODUCTION:

Gravitational waves are ripples in the structure of space-time which propagate as waves and travels outward from the source. One of the vital properties of this wave is that it transports energy as gravitational radiation and this radiation or energy creates the distortion of the medium (Ohanian and Ruffini, 1994). In General theory of relativity, this radiation known as gravity which is connected to space-time curvature (Narlikar, 1978; Max, B. Einstein's, 1964; Clarke, 1929; Hughston and Tod, 1990; Foster, 1994; and Biressa and Pacheco, 2011). Heavy mass creates this curvature (Weinberg, 1972).

Identifying of Gravitational waves

The LIGO (Laser Interferometer Gravitational-Wave Observatory) scientific collaboration announce on 11 February 2016 the identifying of gravitational waves which is known as quadruple wave which uses the space-time as a medium (Ohanian and Ruffini, 1994). By the expansion and contraction of the space it spreads out through the medium. The character of quadruple wave is that if it contracts the space to the horizon, the space will be expansion to the upright at the same time and vice-versa (Caroll, 2019).

By the ultimate slice of a minimum time of the unification, it released over force than fifty times that of all the heavenly body in the notice able united cosmos. The single picks up the high place between the vibration of 35 and 250 Hz. By rotating two dead massive stars a black hole is created which creates distortion over the space time known as gravitational waves (Ohanian and Ruffini, 1994). This wave has two vital and singular characters. Primarily, it is need not to present the substance for creating of the waves by a dual system of zero charged black holes, which would transmit no electromagnetic diffusion. And another is that it can take across any interior body without being dispersed. Light from remote stars may be enclosed out by stellar pollen, while gravitational waves will cross originally unhindered (Ohanian and Ruffini, 1994). By these two modes this waves bear information about astronomical occurrence never before noticed by humans.

Plane-Wave Solutions and the Transverse Trace-less (TT) Gauge

The Linearized field eq. in empty space is given by

$$\Box^2 H^{\mu\upsilon} = 0 \quad (1)$$

With the flat wave solutions,

$$H^{\mu\upsilon} = Re\left[A^{\mu\upsilon} \exp(ik_\alpha x^\alpha)\right] \quad (2)$$

Where $A^{\mu\upsilon}$ is fixed, symmetric, rank-2, and $k^\mu \equiv \eta^{\mu\alpha} k_\alpha$ is a fixed four-vector familiar to the wave vector in the approach of extension and Re stands for "real part". Putting (2) into (1), we have the condition (Wapstra and Nijgh, 1955).

$$k_\alpha k^\alpha = 0 \quad (3)$$

$$\partial_\upsilon H^{\mu\upsilon} = 0, A^{\mu\upsilon} k_\upsilon = 0 \quad (4)$$

Since

$$H^{\mu\upsilon} = H^{\upsilon\mu}$$

Then the amplitude tensor $A^{\mu\upsilon}$ has 10 different (complex) components, but the condition (4) [$A^{\mu\upsilon}$ is orthogonal (transverse) to k_υ] gives four conditions on these, cutting their number down to 6. The gauge condition.

$$\Box^2 \xi^\mu = 0 \quad (5)$$

Suppose a plane wave propagating in the x^3 direction, so that

$$k^\mu = (k, 0,0, k), k_\upsilon = (k, 0,0, -k)$$

Equation (4) implies that $A^{\mu 0} = A^{\mu 3}$, which yields the matrix

$$[A^{\mu\upsilon}] = \begin{bmatrix} A^{00} & A^{01} & A^{02} & A^{00} \\ A^{01} & A^{11} & A^{12} & A^{01} \\ A^{02} & A^{12} & A^{22} & A^{02} \\ A^{00} & A^{01} & A^{02} & A^{00} \end{bmatrix} \quad (6)$$

Let the solution of (6) as

$$\xi^{\mu\upsilon} = -Re\left[i\varepsilon^\mu \exp(ik_\alpha x^\alpha)\right]$$

Where, ϵ^μ are constants? We have

$$\partial_\upsilon \xi^\mu = Re\left[\varepsilon^\mu k_\upsilon \exp(ik_\alpha x^\alpha)\right] \quad (7)$$

In the new gauge (7) becomes

$$H^{\mu'\upsilon'} = Re[A^{\mu'\upsilon'} \exp(ik_{\alpha'} x^{\alpha'})]$$

But

$$H^{\mu'\upsilon'} = (H^{\mu\upsilon} - \partial^\upsilon \xi^\mu - \partial^\mu \xi^\upsilon + \eta^{\mu\upsilon} \partial_\alpha \xi^\alpha$$

From which, using (7), we obtain

$$A^{\mu'\upsilon'} = (A^{\mu\upsilon} - k^\upsilon \varepsilon^\mu - k^\mu \varepsilon^\upsilon + \eta^{\mu\upsilon} (k_\alpha \varepsilon^\alpha)$$

Since $\exp(ik_{\alpha'} x^{\alpha'})$ differs from $\exp(ik_\alpha x^\alpha)$ by only a first-order quantity (Banu *et al*., 2021).
We conveniently choose constants ε^μ as follows:

$$\varepsilon^0 = (2A^{00} + A^{11} + A^{22})\frac{1}{4k}, \varepsilon^1 = (A^{01}/k)$$

$$\varepsilon^2 = (A^{02}/k), \varepsilon^3 = (2A^{00} - A^{11} - A^{22})/4k)$$

So that $A^{0'0'} = A^{0'1'} = A^{0'2'} = 0$ and $A^{1'1} = -A^{2'2'}$

On dropping primes the matrix of amplitude tensor and this wave in this gauge travelling in the x^3direction, the two components A^{11} and A^{12} completely characterize the wave. In this gauge

$$H \equiv H^\mu_\mu = 0$$

$$A^{00} = A^{33} = 0 \text{ and } A^{11} = -A^{22}$$

It then follows that $h = 0$ so that there is no difference between $h_{\mu\upsilon}$ and $\mu H_{\mu\upsilon}$:

Because of $h = H = 0$ the gauge is called traceless, and because of

$$h_{0\mu} = H_{0\mu} = 0$$

It is called transverse.

Gravitational Waves Propagate through Empty Space-time with Light velocity

Maxwell's equations are second order differential equation under a suitable gauge condition (Richard, 1983). We have the Minkowskian metric is of the form (Stephani, 2004; Wapstra and Nijgh, 1955).

$$\eta_{\mu\nu} = \eta^{\mu\upsilon} = (-1,1,1,1) \quad (8)$$

Here, $g_{\mu\nu}$ is poor, if

$$|g_{\mu\nu} - \eta_{\mu\nu}| \ll 1$$

We suppose that it can be expanded as an infinite series

$$g_{\mu v} = \eta_{\mu v} + \lambda_1 g_{\mu v} + \lambda^2{}_2 g_{\mu v} + \cdots$$

Where, λ is some small parameter? If we limit ourselves to the first order term $\lambda_1 g_{\mu v}$ alone, we can write

$$g_{\mu v} \simeq \eta_{\mu v} + h_{\mu v}$$

If we put

$$h^{\mu v} = \eta^{\mu\sigma}\eta^{v\rho}h_{\sigma\rho}$$

Then, from $g_{\mu\lambda}g^{v\lambda}=\delta_\mu^v$, we get

$$g^{\mu v} \simeq \eta^{\mu v} - h^{\mu v}$$

To derive the linearized Einstein equations we have to find the first approximation value of the Ricci tensor, the Ricci scalar, and the Christoffel symbols (Ahmed and Iqbal, 2020). A simple calculation then gives

$$\Gamma^\lambda_{\mu v} = \frac{1}{2}(\partial_\mu h^\lambda_v + \partial_v h^\lambda{}_\mu - \partial^\lambda h_{\mu v}) \qquad (9)$$

So, the Ricci tensor is

$$R_{\mu v}=\frac{1}{2}(\partial_\lambda\partial_\mu h^\lambda_v + \partial_\lambda\partial_v h^\lambda_\mu - \partial^\lambda\partial_\lambda h_{\mu v} - \partial_v\partial_\mu h) \qquad (10)$$

The field equations implies (Weinberg's, 1972).

$$\partial_\lambda\partial_\mu h^\lambda{}_v + \partial_\lambda\partial_v h^\lambda{}_\mu - \partial^\lambda\partial_\lambda h_{\mu v} - \partial_v\partial_\mu h - \eta_{\mu v}\left(\partial_\rho\partial_\lambda h^{\lambda\rho} - \partial^\lambda\partial_\lambda h\right) = 2\kappa T_{\mu v}$$

Introducing the new variables

$$H_{\mu v} = h_{\mu v} - \frac{1}{2}\eta_{\mu v}h$$

We obtain

$$-\partial^\lambda\partial_\lambda H_{\mu v} - \left(\eta_{\mu v}\partial_\rho\partial_\lambda H^{\lambda\rho} - \partial_\lambda\partial_\mu H^\lambda_v - \partial_\lambda\partial_v H^\lambda{}_\mu\right) = 2\kappa T_{\mu v} \qquad (11)$$

We further simplify (11) by a gauge transformation defined by

$$x^{\mu'} \equiv x^\mu + \xi^\mu(x^\alpha) \qquad (12)$$

Now the matrix element is given by

$$\Lambda^{\mu'}_v = \delta^\mu_v + \partial_v\xi^\mu \qquad (13)$$

We find

$$g^{\mu'v'}=\eta^{\mu'v'} - h^{\mu'v'} = \eta^{\mu v} + \partial^\mu\xi^v + \partial^v\xi^\mu - h^{\mu v}$$

Neglecting products of small quantities it leads on rearranging to

$$h^{\mu'v'} = h^{\mu v} - \partial^v\xi^\mu - \partial^\mu\xi^v \qquad (14)$$

Contracting with $\eta_{\mu v}$ it yields

$$h' = h - 2\partial_\mu\xi^\mu \qquad (15)$$

Also,

$$H^{\mu'v'} = H^{\mu v} - \partial^v\xi^\mu - \partial^\mu\xi^v + \eta^{\mu v}\partial_\alpha\xi^\alpha \qquad (16)$$

And

$$\Lambda^{\lambda'}_\mu\Lambda^\mu_{v'} = (\delta^\lambda_\mu + \partial_\mu\xi^\lambda)(\delta^\mu_v - \partial_v\xi^\mu) = \delta^\lambda_v - \partial_v\xi^\lambda + \partial_v\xi^\lambda = \delta^\lambda_v$$

Hence, the inverse matrix element $\Lambda^\mu_{v'} \equiv \frac{\partial x^\mu}{\partial x^{v'}}$ is given by

$$\Lambda^\mu_{v'} = \delta^\mu_v - \partial_v\xi^\mu$$

Thus we obtain

$$\partial_{\alpha'}H^{\mu'v'} = \partial_\alpha H^{\mu\alpha} - \partial^\alpha\partial_\alpha\xi^\mu \qquad (17)$$

On using (16)

$$\partial_\alpha H^{\mu\alpha} = \partial^\alpha\partial_\alpha\xi^\mu \qquad (18)$$

$$\text{and } \partial_{\alpha'}H^{\mu'\alpha'} = 0 \qquad (19)$$

Now we obtain

$$-\partial^{\lambda'}\partial_{\lambda'}H_{\mu'v'}=2\kappa T_{\mu'v'}$$

On dropping primes, we finally obtain for the linearized Einstein field equations the following

$$-\partial^\lambda\partial_\lambda H_{\mu v} = 2\kappa T_{\mu v} \qquad (20)$$

Along with the supplementary (gauge) condition

$$\partial_\lambda H^{\mu\lambda} = 0 \qquad (21)$$

With $\eta_{\mu v} = \eta^{\mu v} = (-1,1,1,1)$ introduce the d' Alembertian is

$$\Box^2 \equiv \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \qquad (22)$$

Then, the linearized Einstein equations (20) becomes

$$\Box^2 H^{\mu v} = -2\kappa T^{\mu v} \qquad (23)$$

The remaining gauge freedom $x^\mu \to x^\mu + \xi^\mu$ preserves the gauge condition provided ξ^μ satisfies

$$\Box^2\xi^\mu = 0 \qquad (24)$$

Evidently equation (23) represents a wave equation with source term

$$-2\kappa T^{\mu v} \equiv -\left(\frac{16\pi G}{c^4}\right)T^{\mu v}$$

In empty space equation (23) reduces to

$$\Box^2 H^{\mu v} = 0 \tag{25}$$

It shows that these waves approach through null space-time having light velocity (Iqbal *et al*., 2020).

Gravitational Waves and Quantum Mechanics

We know earth emits 0.005 watt energy during rotating about the sun. Dirac was able to quantize the field equations and showed that gravitation-quanta or gravitation is the multiple of plank constant h like as photon; although the spin of gravitation is twice number of photon. Now, we would explain a plane wave solution, with wave vector k_μ and helicity ± 2, as consisting of gravitations: quanta with energy momentum vector $p^\mu = \hbar k^\mu$ and rotation quantities in the direction of speed $\pm 2\hbar$ (Gott *et al*., 1974).

We consider that the number consistency of gravitons with helicity ± 2 in a plane wave is

$$N = N_+ + N_- = \frac{\omega}{16\pi\hbar G}\left(e^{\lambda v*}e_{\lambda v} - \frac{1}{2}\left|e^\lambda{}_\lambda\right|^2\right) \tag{26}$$

The gravitational radiation by any system as giving the rate $d\Gamma$ of emitting gravitons of energy $\hbar\omega$ into the solid angle $d\Omega$ is

$$d\Gamma = \frac{dp}{\hbar\omega} = \frac{G\omega d\Omega}{\hbar\pi}\left[T^{\lambda v*}(\vec{k},\omega)T_{\lambda v}(\vec{k},\omega)\right] - \frac{1}{2}\left|T^\lambda{}_\lambda(\vec{k},\omega)\right|^2 \tag{27}$$

Now for usual Lorentz-covariant heavy diffusion relation $\omega^2 = k^2 + m^2$, one gets

$$v_g = 1 - \frac{1}{2}\frac{m^2}{\omega^2} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{m^2}{\omega^2}\right)^2 + \cdots \tag{28}$$

By LIGO the signal of GW150914 is acuminated at vibrations$\omega \approx 6.6 \times 10^{-14} ev$, and the mass of the graviton $m < 1.2 \times 10^{-22} ev$,then we have

$$\left|\Delta v_g\right| < 1.7 \times 10^{-18} \tag{29}$$

By observation equation (29) is mostly overwhelmed for the frequencies which indicate the number of gravity.

CONCLUSION:

We can summarize that gravitation is the manifestation of space-time curvature and the waves created by heavy mass is dynamic with the velocity of light (Stephani, 2004). We have also arises the plane wave solution for quantum theory of gravitational waves by energy momentum tensor which implies the wave like manner and particle manner.

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CONFLICTS OF INTEREST:

The authors declare there are no potential conflicts of interest to publish it under the current issue.

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