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Vibration Analysis of Two-dimensional Functionally Graded Plate with Piezoelectric Layers using the Classical Theory of Plates

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ABSTRACT

In this article, two-dimensional functional materials have been used using the power law in them, which is a good measure to obtain the properties of a composite material of metal and ceramic. At first, the equations of motion were obtained using Hamilton's method and solved by GDQ method, and finally, the accuracy of the obtained answers was compared with the existing articles. In the following, the dynamic model of the sheet with two piezoelectric actuator layers at the top and bottom was investigated and the obtained equations were solved using the Ritz method.

Keywords: Two-dimensional functional sheet vibration, Hamilton's principle, GDQ, and piezoelectric.

INTRODUCTION:

Rectangular plates are one of the mechanical elements that are widely used in various industries such as oil and gas and petrochemical industries. For this reason, investigating the phenomenon of vibrations in this type of plates is a necessary subject, therefore, in this thesis; the vibrations of such plates under the effect of piezoelectric layers that actuate them have been investigated. Examining the vibrations of sheets leads to a series of differential equations. Numerical solution of differential equations is widely used in various fields of engineering. Common methods for solving these equations are FEM, GDQ and RITZ, which are among the numerical methods for solving equations with partial derivatives. In this article, the stability of the sheet with the boundary conditions cccc, ccss, ccsc, cscs, sss and csss has been investigated. The variable of the problem is the piezoelectric thickness and the aim is to find the natural frequency of the sheet along with the piezoelectric. Investigating the piezoelectric effect on plate vibrations and applying the GDQ method to rectangular plates made of two-dimensional FGM materials are new aspects of the work. Research objectives investigating the natural frequencies of two-dimensional FGM sheet under the effect of piezo-electric layers is one of the research objectives.

Hypotheses or research questions 1- The sheet is assumed to be made of two-dimensional graded material. 2- The power law is used to obtain the mechanical properties of the FGM sheet structure. 3- To obtain the stress and strain tensors in the main structure and the piezoelectric layer, the classic sheet model is used. 4- When using the piezoelectric layer, they are used in pairs at the top and bottom of the sheet (the layers are assumed as operators). 5- Poisson's ratio is considered the same. Classical plate theory: In this theory, only strain is ignored and the displacement field inside the plate changes as a linear function of thickness. Hamilton's principle: Hamilton's principle is used as a reference to obtain the equations of motion of the sheet will be GDQ: It is a numerical method in the definition of the derivative and it is used to solve the problem numerically.

Review of Literature

In 2009, Liu (Zhao and Liew, 2009) investigated the free vibrations of a rectangular sheet made of graded functional materials using the Ritz method. The properties of the functional material are considered according to the variable tracking power model in the direction of the sheet thickness. The differential equations of motion of the problem are obtained based on the first-order deformation theory. Finally, the solution of the obtained equations has led to the solution of an eigenvalue problem that has been solved by the Ritz method. In 2012, Akbari and his colleagues (Alashti and Khorsand, 2012) analyzed the vibrations of a cylindrical shell made of graded functional materials based on three-dimensional elasticity with piezoelectric layers and taking into account dynamic and thermal loads, and the equations obtained by the differential quadratic method have been solved and compared with the finite difference method. Akhwan and his colleagues (Akhavan *et al.,* 2009) in 2008 have investigated the exact solution of the buckling analysis of a rectangular sheet under load in an elastic bed. The analysis method is based on the theory of rectangular sheet taking into account the first order shear deformation effect. In reference number (Yas and Aragh, 2010) Yas and his colleagues in 2010 have analyzed the free vibration of a plate reinforced with continuous fiber in an elastic bed. The equations of motion based on three-dimensional elasticity have been solved using the differential quadratic method, and finally the frequencies of the system have been investigated by changing the volume of fiber and composite. In 2009, Nemat-Alla and his colleagues (Nemat-Alla *et al.,* 2009) analyzed the plastic elastic analysis of functionally graded materials under thermal loading. The stress-strain relationships based on the elastic-plastic rule of functionally graded materials under thermal loading have been introduced in this model, and the effects of thermal load on functionally graded material and stress-strain changes in the material have been investigated. In 2009, Malekzadeh (Malekzadeh, 2009) analyzed the vibration of a thick plate with a graded function on an elastic bed. The formulation is based on three-dimensional elasticity and FGM relationships are assumed to be exponential. Shaaban and his colleagues (Sharma and Singh, 2011) in 2011 presented an analytical solution for the free vibration of a thick sheet of functionally graded material in an elastic bed with elastic edges. Governing motion equations based on the first order theory of shear deformation and have been solved numerically and the effects of different parameters have been evaluated. Hashemi (Hashemi *et al.,* 2010) in 2010 presented the free vibrations of a rectangular sheet of functionally graded material using the first-order theory of shear deformation, and a new formula for shear correction of the sheet theory is presented in this paper. Hong *et al*. (Huang *et al.,* 2011) in 2011 have investigated crack vibrations in rectangular thin sheet of functionally graded material. Hu and his colleagues (Thai and Choi, 2011) in 2011 have presented the vibration of a rectangular sheet of functionally graded material in an elastic bed based on the first-order theory of shear deformation. Nezahat and his colleagues (Shirazi *et al.,* 2011) in 2011 investigated the control of active vibrations of a functional rectangular sheet using the fuzzy control method and comparing it with PID controllers and using classical theory and considering a series of local functions of equations obtained the motion and obtained the natural frequencies using the Fourier series.

In 2013, Sadek and his colleagues (Sadek *et al.,* 2003) analyzed the vibrations of a sheet with simple supports based on the feedback control method, which was controlled by sensors and piezoelectric movements, and obtained the equations from the Green's function. The controls are based on displacement and speed feedbacks and solved the equations using a numerical method. In 2013, Rehmat Talebi and his colleagues (Talabi and Saidi, 2013) analyzed the vibrations of complete circular and mounted circular plates made of functional materials due to the effect of two layers of piezoelectric actuators on the top and bottom of the plate using the third order theory of shear and with a new method 5 The main equation of motion is obtained by considering Maxwell's equation. Hashemi and his colleagues (Hosseini Hashemi, 2019) in this paper, the analysis and free vibrations of the targeted relatively thick rectangular plate (FG) with smart layers based on Mendelin theory and providing an exact closed-response solution are investigated. This structure consists of a FG sheet and two piezoelectric layers.

Shariayat and his colleagues (Shariayat, 2018) investigated the effects of using piezoelectric sensor and actuator layers on the vibrations of the quad-rangular FGM sheet, studied free vibrations, forced vibrations and active control of transient vibrations in this regard. Graded functional material Sheets are structures whose initial shape is flat and their thickness is very small. The behavior of the sheets can be distinguished depending on the type of material and the structure of their formation. Graded functional materials are materials whose mechanical properties change gently and continuously. In recent years, with the development of electric motors, turbines and reactors, it seems necessary to use materials that have high thermal and mechanical resistance. And in this chapter, the introduction of the graded functional materials is discussed. Then, how to make and uses of this material will be discussed. This class of materials is introduced here. Among the main applications of these graded functional materials are the use of nuclear reactors (components of the inner wall of the reactor), use in chemical industries (membranes and catalysts) use in medical engineering (implantation of artificial teeth, bones or artificial organs) and mentioned other new technologies such as ceramic motors and coating against corrosion and heat.

Also, these materials can be widely used in making plates and shells of tanks, reactors, turbines and other components of machines exposed to high heat. Because these parts are highly prepared for failure due to thermal buckling. Another advantage of FG materials compared to layered composite materials is the lack of discontinuity at the junction of layers, because as mentioned in FGM materials, the combination of elements is continuous and gradual. Among the other advantages of graded functional materials, we can mention their use in the construction of thermal insulation coatings, which are widely used. Graded functional materials are composite materials that are microscopically inhomogeneous and their structural characteristics such as the type of distribution, the size of phases, gradually change from one surface to another, and this gradual change leads to a gradual change in their properties. These materials are generally made of a mixture of ceramic with metal or a combination of different metals (with different thermal coefficients).

The ceramic material has high temperature resistance due to its low thermal conductivity, and the metal material prevents breakage or cracking due to thermal stress due to its malleability. In the simplest FGMs, two different material components are continuously changed from one to the other. The transverse distribution of electric potential satisfies the Maxwell equation as well as the electrical boundary conditions for both closed and open circuit piezoelectric layers. (Farsangi and Saidi, 2012) In the micro-electro-mechanical systems (MEMS), Coulomb's force plays a major role as a mechanism in sensing and actuation. MEMS devices due to their high sensitivity are widely used in different fields of science (Kazemi *et al.,* 2019). The free vibration analysis of two different configurations of functionally graded material (FGM) plates and cylinders is proposed. The first configure-tion considers a one-layered FGM structure. The second one is a sandwich configuration with external classical skins and an internal FGM core. Low and high order frequencies are analyzed for thick and thin simply supported structures. Vibration modes are investigated to make a comparison between results obtained via the 2D numerical methods and those obtained by the means of the 3D exact solution (Brischetto *et al.,* 2016; Akter *et al*., 2023)

METHODOLOGY:

Validation of two-dimensional functional sheet vibration with existing articles to validate the results of this research, we first run the written program in the special case of one-dimensional graded material and without piezoelectric. Then we compare the obtained results with references 7, 8 and 11. In these references, the function sheet one dimension is made of aluminum metal and aluminum oxide ceramic with the following specifications.

Table 1: specifications of one-dimensional functional grade material (AL/AL_2O_3).

material	E (GPA)	$\rho(kg/m^3)$
AL	70 e9	2702
AL_2O_3	380 e9	3800

The results of this review are as follows

Table 2: Comparison of natural four-frequency with dimension of present research with existing articles for one-dimensional functional material SSSS) و $\frac{a}{b}=1$)و($\frac{a}{h}=10$) و($nx=0, nz=1$)

	ω_1	ω_2	ω_3	ω_4
Present	4492	11092	11128	16880
X. Zhao[11]	4347.4	10416	10416	15936
Error (%)	3.326126	6.490015	6.835637	5.923695
H. Matsunaga[7]	4427	10630	10630	16200
Error (%)	1.468263	4.34619	4.684854	4.197531

Table 2 Comparison of the main natural frequency with the dimension of the current research with existing articles for the one-dimensional functional material AL/AL_2O_3 for boundary conditions SSSS) و $\frac{a}{b}=1$)و($\frac{a}{h}=10$)و($nx=0$)

	nz				
	0	0.5	1	4	10
Present	5950	5225	4492	4070	3855
H. Matsunaga [7]	5777	4917	4427	3811	3642
error	2.994634	6.263982	1.468263	6.796117	5.848435
Sh. H.Hashemi [8]	5769	4920	4454	3825	3627
error	3.137459	6.199187	8.53166	6.405229	6.286187
X. Zhao [11]	5676.3	4820.9	4347	-	3592.3
error	4.821803	8.382252	3.335634	-	7.312864

Investigation of two-dimensional functional sheet vibration (without piezoelectric) $h_p=0$

Table 3: Specifications of two-dimensional graded material.

material	E (GPA)	$\rho(kg/m^3)$
$AL(m_1)$	70	2702
$steel\,(m_2)$	200	7800
AL_2O_3	380	3800
$Ni\,(c_2)$	205	8900

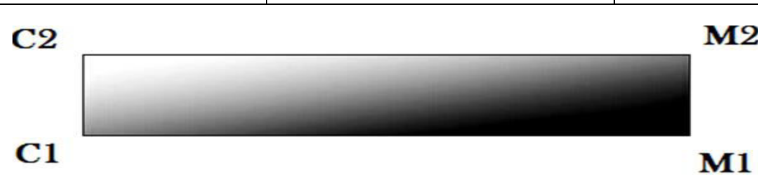

Table 4: Checking the vibration of two-dimensional graded material with support conditions SSSS.

ssss			
$\frac{a}{h}$	$nz=0.5$	$nz=1$	$nz=4$
	nx	nx	nx

	0	0.5	1	4	10	0	0.5	1	4	10	0	0.5	1	4	10
10	2973	3229.3	3344.1	3536.3	3605.6	3021.9	3415.2	3590	3880.06	3980.7	3251.4	3915.3	4190	4640.2	4790.5
20	1495.623	1624.56	1682.312	1779.002	1813.865	1520.223	1718.08	1806.017	1951.937	2002.566	1635.678	1969.665	2107.858	2334.339	2409.951
30	998.064	1084.185	1122.905	1186.995	1209.694	1014.754	1146.939	1205.688	1303.158	1336.538	1092.196	1314.507	1406.636	1558.181	1608.252
40	748.548	813.1388	842.1788	890.2463	907.2705	761.0655	860.2043	904.266	977.3685	1002.404	819.147	985.8803	1054.977	1168.636	1206.189
50	598.8384	650.511	673.743	712.197	725.8164	608.8524	688.1634	723.4128	781.8948	801.9228	655.3176	788.7042	843.9816	934.9086	964.9512
80	374.274	406.5694	421.0894	445.1231	453.6353	380.5328	430.1021	452.133	488.6843	501.2018	409.5735	492.9401	527.4885	584.3179	603.0945
100	299.4192	325.2555	336.8715	356.0985	362.9082	304.4262	344.0817	361.7064	390.9474	400.9614	327.6588	394.3521	421.9908	467.4543	482.4756

The resulting graphs from this table are as follows

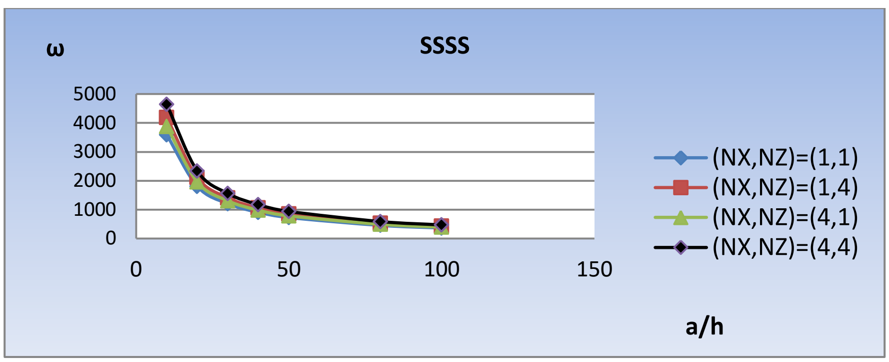

Fig. 1: The original natural frequency with 2D functional sheet dimension with SSSS boundary conditions.

Fig. 2: The original natural frequency with the dimension of the two-dimensional functional sheet with boundary SSSS و ($a/h = 10$) conditions.

According to the graph (1), it can be seen that with the increase of the ratio or the decrease of the thickness, the natural frequency of the two-dimensional functional sheet decreases. 2- According to the diagram (2), it can be seen that it increases with increasing (constant) frequency. 3- According to the graph (2), it can be seen that it increases with the increase (constant) of the frequency. This increase is faster for values less than one, and for values greater than one, this increase is slower. 4- According to the diagram (1) by comparing the two cases, it can be seen that the increase has a greater effect on the frequency increase than the increase. 5- According to the diagram (1), it can be seen that the frequency reduction is faster before and after this ratio, the frequency reduction speed becomes lower.

Table 5: Checking the vibration of two-dimensional graded material with SCSC support conditions of the main natural frequency with the dimension of the two-dimensional functional sheet with SCSC boundary conditions.

scsc															
$\frac{a}{h}$	$nz=0.5$					$nz=1$					$nz=4$				
	nx					nx					nx				
	0	0.5	1	4	10	0	0.5	1	4	10	0	0.5	1	4	10
10	4428.9	4796.3	4975	5295.2	5385.4	4500.3	5070.1	5345	5820	5955.7	4835.1	5805	6240	6975.3	7175
20	2228	2412.824	2502.721	2663.8	2709.176	2263.918	2550.562	2688.853	2927.806	2996.071	2432.343	2920.26	3139.091	3508.84	3609.452
30	1486.324	1609.622	1669.594	1777.052	1807.322	1510.286	1701.509	1793.764	1953.173	1998.713	1622.644	1948.139	2094.123	2340.787	2407.906
40	1117.621	1210.333	1255.428	1336.23	1358.991	1135.639	1279.426	1348.796	1468.661	1502.905	1220.125	1464.876	1574.647	1760.123	1810.592
50	894.1684	968.3442	1004.423	1069.069	1087.28	908.5836	1023.623	1079.123	1175.023	1202.42	976.1777	1171.995	1259.819	1408.211	1448.589
80	558.8902	605.253	627.8034	668.21	679.5925	567.9002	639.8042	674.4943	734.4353	751.5596	610.1492	732.5425	787.4358	880.1867	905.425
100	447.1479	484.2411	502.2829	534.6107	543.7175	454.3565	511.8843	539.6386	587.5953	601.2957	488.1584	586.0809	629.9991	704.2057	724.398

The resulting graphs from this table are as follows

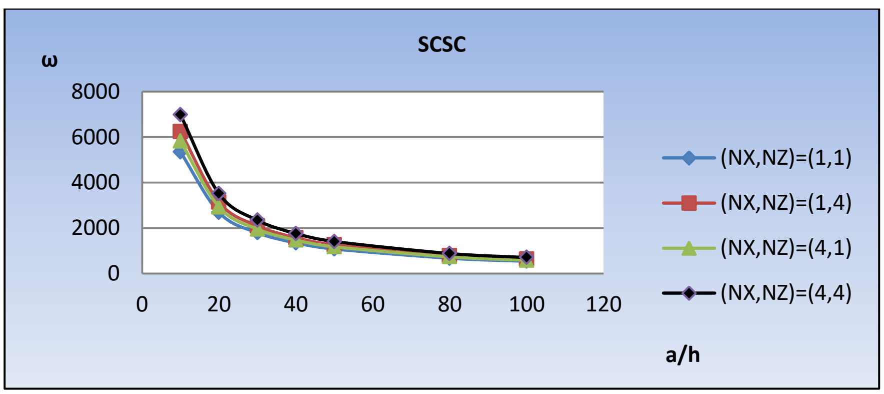

Fig. 3: The original natural frequency with the dimension of the two-dimensional functional sheet with boundary SCSC conditions.

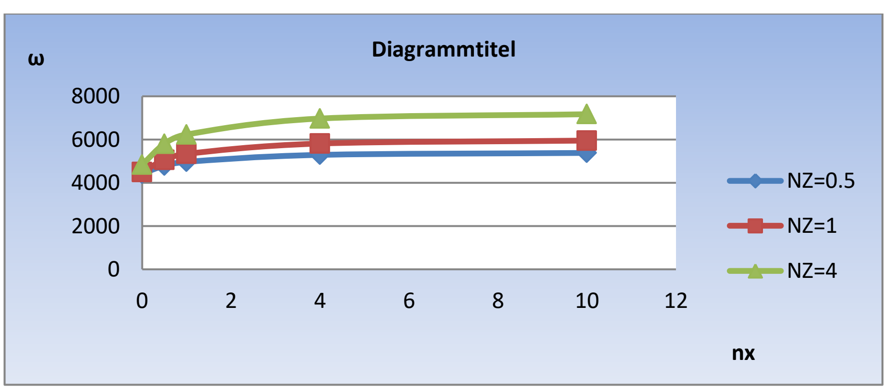

Fig. 4: Checking the vibration of two-dimensional graded material with CCCS support conditions along with piezoelectric.

Table 6: main natural frequencies of two-dimensional functional sheet with thickness ( ) under the effect of piezoelectric layer with thickness ( ) with CCCS boundary conditions.

CCCS(h=0.1)					
h_p	$nz=0$	$nz=1$		$nz=4$	
	nx	nx		nx	
	0	1	4	1	4
0	4945	5840	6365	6800	7610
0.01	4830	5530	5990	6210	6910
0.02	4840	5425	5815	5955	6550
0.03	4940	5455	5780	5890	6395
0.04	5110	5570	5845	5950	6370
0.05	5325	5745	5970	6085	6435
0.06	5575	5960	6150	6270	6565
0.07	5845	6205	6335	6495	6735
0.08	6130	6470	6600	6735	6940
0.09	6425	6745	6855	6995	7170
0.1	6725	7025	7115	7265	7410

The graph resulting from this table is as follows

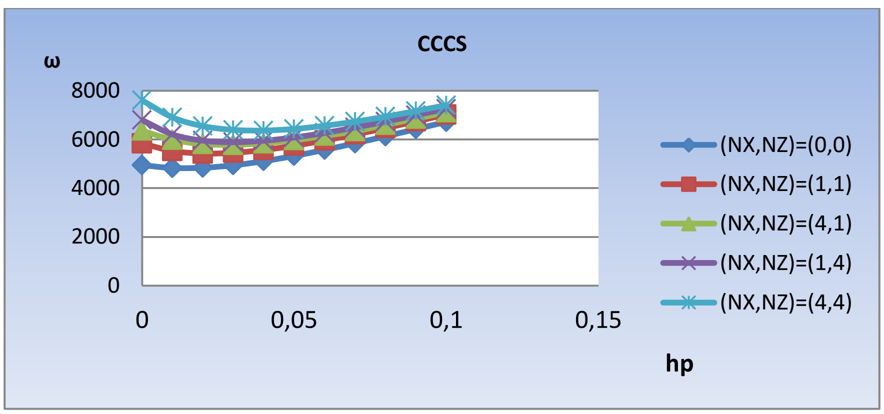

Fig. 5: The main natural frequencies of the two-dimensional functional sheet with thickness ( ) under the effect of piezoelectric layer with thickness ( ) with CCCS boundary conditions.

In the case of increasing the piezoelectric thickness ( ), the frequency decreases until the thickness and the frequency increases from this thickness. 2- In the case of ( ) increasing the piezoelectric thickness ( ), the frequency decreases ( ) to the thickness ( ) and the frequency increases from this thickness. 3- In the case of increasing the piezoelectric thickness, the frequency decreases to the thickness ( ) and the frequency increases ( ) from this thickness. 4- In the case of increasing the piezoelectric thickness, the frequency decreases to the thickness and the frequency increases from this thickness. 5- In the case of increasing the piezoelectric thickness ( ), the frequency decreases to the thickness ( ) and the frequency increases from this thickness. 6- By increasing the thickness of the piezo ( ), all the graphs are closer to each other and finally one be. 7- As the effect of piezoelectric increases in the initial reduction of frequencies, it increases. For example, in the case of frequency reduction, it is equal to 5.3% and for the case of equal to 8.6% is. 8- As the effect of piezoelectric increases in the initial reduction of frequencies, it increases. For example, in the mode of reducing the frequency in the mode equal to 5.3% and for the mode equal to 5.8% is. 9- The effect of piezoelectric in the initial reduction of frequencies is greater than For example, in the mode of reducing the frequency in the mode equal to 5.8% and for the mode equal to 8.6% is 6-3-5- Investigating the vibration of two-dimensional graded material with CCCC support conditions along with piezoelectric.

Table 7: main natural frequencies of two-dimensional functional sheet with thickness ( ) under the effect of piezoelectric layer with thickness ( ) with CCCC boundary conditions.

CCCC(h=0.1)					
h_p	$nz=0$	$nz=1$		$nz=4$	
	nx	nx		nx	
	0	1	4	1	4
0	5610	6655	7155	7750	8520
0.01	5475	6305	6745	7080	7760
0.02	5488	6180	6560	6790	7370
0.03	5600	6205	6525	6715	7205
0.04	5790	6330	6600	6775	7185
0.05	6035	6525	6750	6920	7265
0.06	6315	6765	6950	7125	7415
0.07	6620	7040	7195	7370	7610
0.08	6945	7335	7460	7640	7845
0.09	7275	7640	7745	7930	8095
0.1	7610	7955	8040	8225	8365

The graph resulting from this table is as follows

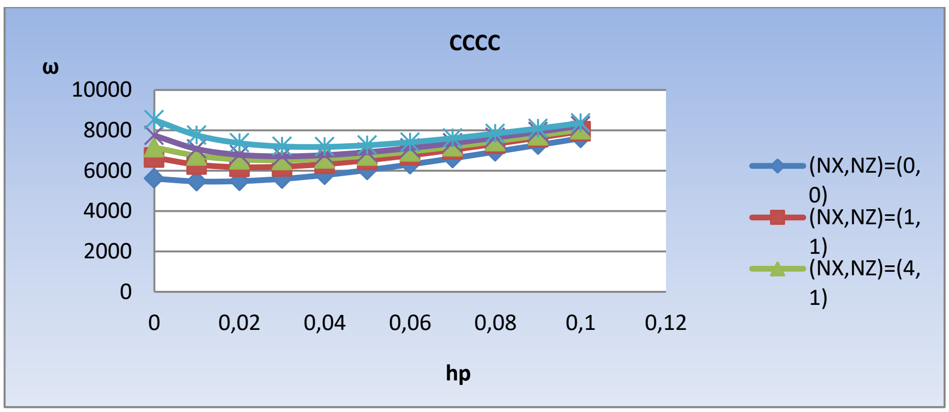

Fig. 6: The main natural frequencies of the two-dimensional functional sheet with thickness ( ) under the effect of piezoelectric layer with thickness ( ) with CCCC boundary conditions.

Comparing the vibration of two-dimensional graded material with two piezoelectric layers with SSSS, SCSC, SSSC, SSCC, SCCC and CCCC boundary conditions.

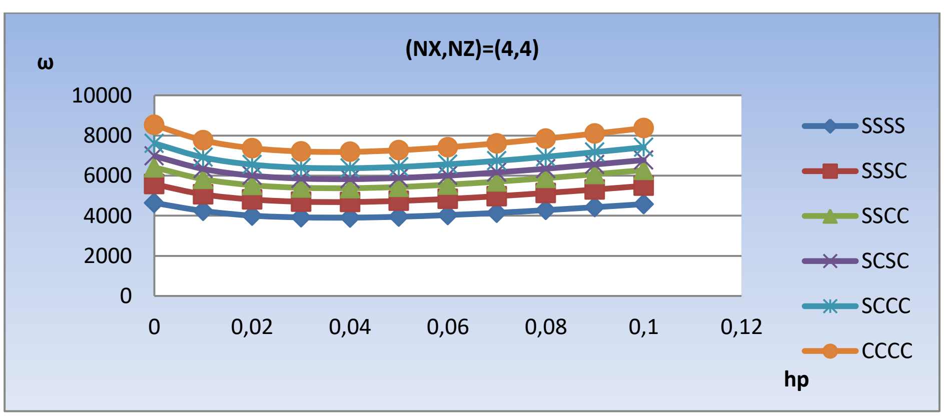

Fig. 7: The original natural frequency with the dimension of a two-dimensional functional sheet with two piezoelectric layers with different boundary conditions in the state.

RESULTS:

According to the diagram above, in a specific state (here for), the percent increase and decrease in frequency caused by changes in the piezoelectric thickness is almost constant and uniform. So that all six lines in the above diagram move parallel to each other. Investigating the causes of errors 1- The classical theory of sheets is only valid for thin sheets. In this theory, shear force and rotational inertia are zero. 2- To consider zero and to simplify the equations. 3- Using the numerical method (GDQ) to solve equations: the causes of errors in this part include two parts becomes: a: The smaller the number of points in the grid of the sheet, the greater the error. b: In this research, the method is used to apply the boundary conditions. In this method, we apply the boundary conditions regardless of the delta distance from the edges.

$$\left.\begin{aligned} u &= u_0 - z\frac{\partial w_0}{\partial x} \\ v &= v_0 - z\frac{\partial w_0}{\partial y} \\ w &= w_0 \end{aligned}\right\} \rightarrow \left\{\begin{aligned} u &= -z\frac{\partial w_0}{\partial x} \\ v &= -z\frac{\partial w_0}{\partial y} \\ w &= w_0 \end{aligned}\right.$$

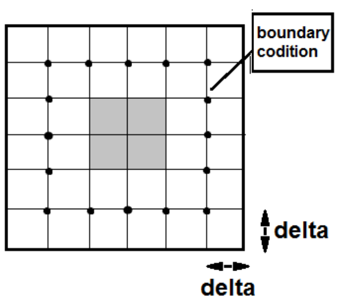

Fig. 8: Boundary condition in the state.

CONCLUSION:

According to the tables and charts, it was observed that: For all boundary conditions and different power coefficients, natural frequency decreases with increasing ratio ( ). They increase with increasing and frequencies. It should be noted that the frequency changes depend on the arrangement of metals and ceramics in the two-dimensional graduated functional material. As expected, in a certain case, the thickness of the functional sheet and the power coefficients are ordered from higher to lower according to the boundary conditions as follows: from CCCC, CCCS, SCSC, CCSS, SSSC and SSSS. The effect of the presence of two piezoelectric layers on the top and bottom of the sheet in the (close-circuit) mode is such that with the increase in thickness, the piezoelectric frequencies decrease up to a certain thickness and then increase. The piezo effect in a certain state of plate thickness and power coefficients is completely similar for different boundary conditions. This means that the frequencies decrease and then increase at a certain rate.

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CONFLICTS OF INTEREST:

Conflicts of interest are declared obviously in the manuscript. Authors also state separately that they have all read the manuscript and have no conflict of interest.

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