Dust Ion Acoustic Solitary Waves in Multi-Ion Dusty Plasma System

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ABSTRACT

A theoretical work has done to observe the existence of dust ion-acoustic (DIA) solitary waves (SWs) in a multi-ion dusty plasma system consisting of inertial positive and negative ions, Maxwell’s electrons, and arbitrary charged stationary dust. In this short communication, our research declares that with these components the derivation of Korteweg-de Vries (K-dV) and mixed K-dV (mK-dV) is possible. Here reductive perturbation method has been employed in all these approaches. The first K-dV equation has been derived which gave both bright and dark solitons but for a very limited region. Then the mK-dV equation has been derived that gave bright soliton for a large region, but no dark soliton has been observed.

Keywords: Dusty plasma, Ion acoustic wave, Multi-ion, DIA, Space plasma system, K-dV, and mK-dV.

INTRODUCTION:

Dusty plasma is normal electron-ion plasma with an added highly charged element of small micron or sub-micron sized extremely massive charged gritty (dust grains). Shukla and Silin, (1992) have theoretically shown the low-frequency dust-ion-acoustic waves in a dusty plasma system. Barkan et al. (1995) have experimentally verified the existence of dust-ion-acoustic wave in dusty plasma. These waves differ from usual ion-acoustic waves (Lonngren, 1983) due to the conservation of equilibrium charge density \(n_{e0}+n_{d0}Z_{d}e+n_{i0}e=0\), and the strong inequality, \(n_{e0} \ll n_{d0}\), where \(n_{s0}\) is the particle number density of the species \(s\) with \(s = e (i) d\) for electrons (ions) dust, \(Z_{d}\) is the figure of electrons residing onto the dust grain side, and \(e\) is the magnitude of an electronic charge. The nonlinear structures related with the DIA waves are particularly solitary waves (Bharuthram and Shukla, 1992; Nakamura and Sharma, 2001); shock waves (Nakamura et al., 1999; Luo, 2000; and Mamun and Shukla, 2002), etc. These waves have also had a great deal of interest to understand the localized electrostatic perturbations in galactic space (Geortz, 1989; Fortov, 2005), and laboratory dusty plasmas (Shukla and Mamun, 2002; Nakamura and Sharma, 2001, and Barkan et al., 1996).

Dusty plasmas create a fully modern interdisciplinary area with direct link to astrophysics, nanoscience, fluid mechanics, and material science as specified through experimental, theoretical, analytical, and arithmetical studies. All of these works (Shukla and Mamun, 2002; Bharuthram and Shukla, 1992; Nakamura and Sharma, 2001; Nakamura et al., 1999; Luo, 1995; Mamun, 2009) are limited to planar (1D) geometry and are subjected to some critical value.
A few works have also been done on finite amplitude DIA solitons and shock structures (Luo, 1995), where K-dV or Burgers equations are used, which are not valid because, the latter gives infinitely large amplitude structures which break down the validity of the reductive perturbation method) for a parametric regime corresponding to $A = 0$ or $A \sim 0$ (where $A$ is the coefficient of the nonlinear term of the K-dV or Burgers equation) (Luo, 1995).

Here, $A \sim 0$ means $A$ is not equal to 0, but $A$ is around 0. In our present work we have been able to show the bright and dark solitons for a large region of multi-ion dusty plasma system in an adiabatic state.

The manuscript is prepared as follows; the model equations are given in Sec. 2, the K-dV equation is derived in Sec. 3, the mK-dV equation is derived in Sec. 4, then results and discussion are given in Sec. 5, and conclusion is given in Sec. 6.

**Model Equations**

The dynamics of the one-dimensional multi-ion DIA waves are governed by:

\[ \partial n_s / \partial t + \partial \rho_s / \partial x = 0, \]
\[ \partial \rho_s / \partial t + u_s \partial \rho_s / \partial x = \partial \psi / \partial x - (\delta / n_s) (\partial \rho_i / \partial x), \]
\[ \partial n_i / \partial t + u_i \partial n_i / \partial x = \partial \psi / \partial x - \delta / n_i \partial \rho_i / \partial x, \]
\[ \epsilon \partial \psi / \partial x - 1/n_s \partial \rho_i / \partial x = 0, \]
\[ \partial \rho_i / \partial t + u_i \partial \rho_i / \partial x + \gamma \rho_s \partial u_i / \partial x = 0, \]
\[ \partial^2 \psi / \partial x^2 = [1-\mu_s + \mu_d] \exp +\mu_s n_n -\mu_d n_d, \]

where $n_s$ is the number density with $s = n(i) e(d)$ of negative ion (positive ion) electron (stationary dust), $u_s$ is the fluid speed of $s$, $m_j$ is the positive (when $j = i$) or negative (when $j = n$) ion mass, $Z_d$ is the number of electron occupy on the dust grain side, $\epsilon$ = magnitude of the electron-charge ($q$), $\phi$ is the electrostatic wave potential; $n_{d0}$, $(n_j0)$, and $n_d0$ are the equilibrium value of $n_j$, $(n_j)$, and $n_d$ respectively i.e. $n_{d0}$, $(n_j0)$, and $n_d0$ are the number density normalized by $n_{d0}$, $(n_j0)$, and $n_d0$ respectively, $\rho_i$ is the pressure of species $i$, $\gamma$ is an adiabatic index, $x$ is the space variable, and $t$ is the time variable.

**K-dV Equation**

For the DIA K-dV equation we introduce the stretched coordinates:

\[ \zeta = \epsilon^{1/2} (x-V_p t), \]
\[ \tau = \epsilon^{1/2} t, \]

Where, $V_p$ is the wave phase speed ($\omega/k$), and $\epsilon$ is a smallness parameter ($0 < \epsilon < 1$).

To get the dispersion relation, we expand $n_s$, $u_s$, $p_s$, and $\phi$ with $s$ be the charged species like positive and negative ion, electron in power series of $\epsilon$, to their equilibrium and perturbed parts,

\[ n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \epsilon^3 n_s^{(3)} + \cdots, \]
\[ u_s = 0 + \epsilon u_s^{(1)} + \epsilon^2 u_s^{(2)} + \epsilon^3 u_s^{(3)} + \cdots, \]
\[ p_s = 0 + \epsilon p_s^{(1)} + \epsilon^2 p_s^{(2)} + \epsilon^3 p_s^{(3)} + \cdots, \]
\[ \psi = 0 + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \epsilon^3 \psi^{(3)} + \cdots, \]

Where $n_s^{(1)}$, $u_s^{(1)}$, $p_s^{(1)}$, and $\psi^{(1)}$ are the perturbed part of $n_s$, $u_s$, $p_s$, and $\psi$ respectively.

Combining above equations, we get -

\[ V_p = (-b \pm \sqrt{(b^2 - 4ac)/2a})^{1/2}, \]

Where,

\[ \alpha = (1-\mu_s + \mu_d), \]
\[ \beta = \mu_s + \mu-\alpha \gamma \delta_n - \alpha \mu \gamma \delta_n, \]
\[ c = \alpha \gamma \delta_n \gamma \delta_n - \gamma \delta_n \delta_n. \]

Equation (12) represents linear dispersion relation.

The next higher order of $\epsilon$ can be simplified as an equation of the form:

\[ \partial \psi / \partial t + A \psi \partial \psi / \partial \zeta + \beta \partial^3 \psi / \partial \zeta^3 = 0, \]

Where,

\[ A = Y/X, \]
\[ \beta = 1/ X, \]
\[ X = (2V_p/d_s^2) (2V_p \mu_s/d_t), \]
\begin{align*}
Y &= -c_1/d_3^3 + \mu_n c_2/d_1^3 + \mu_c c_3/d_1^3, \\
c_1 &= V_p^4 + 2 V_p^2 \gamma \delta_n - 3 V_p^2 + \gamma^2 \delta_n^2 - 2 \gamma \delta_n + \gamma^2 \delta_n, \\
c_2 &= V_p^4 \mu_n - 2 \mu V_p^2 \gamma \delta_n + 3 \mu V_p^2, \\
c_3 &= \gamma^2 \delta_n^2 + 2 \gamma \delta_n - \gamma^2 \delta_n.
\end{align*}

Equation (13) is known as K-dV equation. We get stationary localized solution of (13) by introducing a transformation \( \xi = \zeta - U_0 \tau \):

\begin{equation}
\psi = \psi_m \text{sech}^2 \left[ (\zeta - U_0 \tau)/\delta \right],
\end{equation}

where the amplitude \( \psi_m \) and the width \( \delta \) are given by \( \psi_m = 3U_0/A \), and \( \delta = \sqrt{4\beta/U_0} \), respectively.

**Fig 1:** Bright and dark K-dV solitons.

Equation (14) is the solution of K-dV equation. This represents a solitary wave. We observed that the Fig 1 shows the existence of bright and dark K-dV solitons with mass number density (\( \mu \)).

**mK-dV Equation**

For the third order calculation a new set of stretched coordinates is applied:

\begin{equation}
\zeta = \epsilon (x - V_p t), \quad \tau = \epsilon^3 t,
\end{equation}

Using (15) we can find the same values of \( n_i^{(1)}, n_n^{(1)}, n_e^{(1)}, u_i^{(1)}, u_c^{(1)}, u_n^{(1)}, p_i^{(1)}, p_c^{(1)}, p_n^{(1)} \), and \( V_p \) as like as that in K-dV.

To the next order approximation of \( \epsilon \), we obtain a set of equations, which, after using the values \( n_i^{(1)}, n_n^{(1)}, n_e^{(1)}, \) and \( V_p \), can be simplified and applying the condition, \( \psi \neq 0 \) (so, it’s coefficient is zero), we get,

\begin{equation}
\frac{1}{2} \{ A(\psi^{(1)})^2 \} = 0
\end{equation}

For the next higher order of \( \epsilon \), we obtain an equation:

\begin{equation}
\ddot{\psi} + \alpha \beta \psi^2 \frac{\partial^2 \psi}{\partial \xi^2} + \beta \frac{\partial^3 \psi}{\partial \xi^3} = 0,
\end{equation}

where,

\begin{align*}
\alpha &= F(-a_1^2 + 15/2 - 21 \gamma \delta_n/2a_1 - 5 \gamma^2 \delta_n/2a_1^2 - 3 \gamma^3 \delta_n^2/a_1 - 2 \gamma \delta_n/2a_2 + G(a_2^2 - 15/2 - 21 \gamma \delta_n/2a_2 + G(-5 \gamma^2 \delta_n^2/a_2 - 3 \gamma^3 \delta_n^2/a_2^2 - 3 \gamma^2 \delta_n/a_2^2)), \\
\beta &= V_p a_1 a_2^2 / (-2 \mu \mu_n a_2^2 V_p^2 - 2 \gamma \delta a_1^2),
\end{align*}

where,

\( F = \mu_n a_1^3, \quad G = 1/a_2^3, \quad a_1 = (\gamma \delta_n - \mu V_p^2), \) and \( a_2 = (V_p^2 - \gamma \delta). \)

Equation (17) is known as mK-dV equation. The stationary localized solution of (17), obtained by introducing a transformation \( \zeta = \zeta - U_0 \tau \), is, therefore, directly given by

\begin{equation}
\psi = \psi_m \text{sech} \left[ (\zeta - U_0 \tau)/\Delta \right],
\end{equation}

where the amplitude \( \psi_m \) and the width \( \delta \) are given by \( \psi_m = \sqrt{6U_0/\alpha \beta} \), and \( \delta = \sqrt{\gamma/\psi_m} \), where the amplitude \( \psi_m \) and the width \( \Delta \) are given by \( \psi_m = \sqrt{6U_0/\alpha \beta}, \Delta = 1/(\sqrt{\gamma} \psi_m) \), and \( \gamma = \alpha/6 \).

**Fig 2:** Bright mK-dV soliton.

Fig 2 gives the existence of mK-dV solitons. We get only one type of solitons, bright solitons.
RESULTS AND DISCUSSION:
Dust ion acoustic K-dV and mK-dV solitons have been investigated in a multi-ion dusty plasma system where we observed;

1. The positive and negative K-dV solitons are observed.
2. The width and amplitude of the K-dV solitons varies with polarity changes.
3. Existence of positive mK-dV solitons is observed.

CONCLUSION:
Present investigation are valid for tiny amplitude DIA K-dV, and mK-dV solitons. The first K-dV equation has been derived which gave both bright and dark solitons but for a very limited region. Though we have considered positive and negative ions, Maxwell’s electrons, and arbitrarily charged stationary dust, our model is applicable for small amplitude waves only.

ACKNOWLEDGEMENT:
We are so much acknowledged to Prof. Dr. Md. Kamal-Al- Hassan and Asst. Prof. Dr. Farah Deeba for their guidance to this research.

CONFLICTS OF INTEREST:
The authors declare that they have no competing interests with respect to the research.

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https://doi.org/10.1063/1.873345

https://doi.org/10.1088/0031-8949/45/5/015


**Citation:** Islam KA, Deeba F, and Hassan MKA. (2019). Dust ion acoustic solitary waves in multi-ion dusty plasma system with adiabatic thermal change, *Aust. J. Eng. Innov. Technol.*, 1(5), 1-5. 
https://doi.org/10.34104/ajeit.019.0105