Dust Ion Acoustic Solitary Waves in Multi-Ion Dusty Plasma System with Adiabatic Thermal Change

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ABSTRACT
This theoretical work has done on the behavior of dust ion acoustic (DIA) solitary waves (SWs) in an adiabatic plasma system consisting of inertial positive and negative ions, Maxwell’s electrons, and arbitrary charged stationary dust. The dust particles have been regarded as either positively or negatively charged in order to perceive the effects of dust polarity on the DIA SWs. Through this work the changes in the main properties of these waves with adiabatic state have been observed. At first, a detail mathematical derivation has done on the linear properties as well as the dispersion relation in the multi-ion dusty plasma system. In order to perceive the properties of SWs two different approaches Korteweg-de Vries (K-dV) and mixed K-dV (mK-dV) has been made. Here reductive perturbation slant has been employed in all these approaches. First K-dV equation has been derived which let to analyze both bright and dark solitons but for a very limited region. Then mK-dV equation has been derived that let analyze bright soliton for a large region.

Keywords: Dusty Plasma, Ion Acoustic Wave, Multi-ion, Space Plasma, K-dV, and mK-dV.

1. INTRODUCTION
Dusty plasma is normal electron-ion plasma with an added highly charged element of small micron or sub-micron sized extremely massive charged gritty (dust grains). Shukla and Silin (1992) have theoretically shown the low-frequency dust-ion-acoustic waves in a dusty plasma system. Barkan \textit{et al.}, (1995) have experimentally verified the existence of dust-ion-acoustic wave in dusty plasma. These waves differ from usual ion-acoustic waves (Lonngren, 1983) due to the conservation of equilibrium charge density $n_{ei}+n_{i0}Z_d e n_{d0} = 0$, and the strong inequality, $n_{e0} \ll n_{i0}$, where $n_{s0}$ is the particle number density of the species $s$ with $s = e$ (i) d for electrons (ions) dust, $Z_d$ is the figure of electrons residing onto the dust grain side, and $e$ is the magnitude of an electronic charge. DIA wave’s linear properties are now prudent understood (Shukla and Mamun, 2002; Barkan \textit{et al.}, 1996; and Shukla and Rosenberg, 1999). The nonlinear structures related with the DIA waves are particularly solitary waves (Bharuthram and Shukla, 1992; Nakamura and Sharma, 2001); shock waves (Nakamura \textit{et al.}, 1999; Luo, 2000; and Mamun and Shukla, 2002), etc. These waves have also had a great deal of interest to understand the localized electrostatic perturbations in galactic space (Geortz, 1989; Fortov, 2005), and laboratory dusty plasmas (Shukla and Mamun, 2002; Nakamura and Sharma, 2001, and Barkan \textit{et al.}, 1996).
Dusty plasmas create a fully modern interdisciplinary area with direct link to astrophysics, nanoscience, fluid mechanics, and material science as specified through experimental, theoretical, analytical, and arithmetical studies. All of these works (Shukla and Mamun, 2002; Bharuthram and Shukla, 1992; Nakamura and Sharma, 2001; Nakamura et al., 1999; Luo, 1995; Mamun, 2009) are limited to planar (1D) geometry and are subjected to some critical value. A few works have also been done on finite amplitude DIA solitons and shock structures (Luo, 1995), where K-dV or Burgers equations are used, which are not valid because, the latter gives infinitely large amplitude structures which break down the validity of the reductive perturbation method) for a parametric regime corresponding to \( A = 0 \) or \( A \sim 0 \) (where \( A \) is the coefficient of the nonlinear term of the K-dV or Burgers equation) (Luo, 1995). Here, \( A \sim 0 \) means \( A \) is not equal to 0, but \( A \) is around 0. In our present work, we have been able to show the bright and dark solitons for a large region of multi-ion dusty plasma system in an adiabatic state.

The manuscript is prepared as follows; the model equations are given in Sec. 2, the K-dV equation is derived in Sec. 3, the mK-dV equation is derived in Sec. 4, then results and discussion are given in Sec. 5, and conclusion is given in Sec. 6.

2. Model Equations

The dynamics of the one-dimensional multi-ion DIA waves are governed by:

\[
\frac{\partial n_i}{\partial t} + \frac{\partial u_i}{\partial x} = 0, \quad (1) \\
\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \psi}{\partial x} - (\delta/n_i) \left( \frac{\partial p_i}{\partial x} \right), \quad (2) \\
\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} = 1/\mu (\partial \psi/\partial x - \delta_j/n_j \frac{\partial p_j}{\partial x}), \quad (3) \\
\frac{\partial^2 \psi}{\partial x^2} = [1-\mu_n + \mu_d] \exp^\psi + \mu_n n_n - \mu_d n_d, \quad (4) \\
\frac{\partial \psi}{\partial x} - u_j \frac{\partial p_j}{\partial x} + \gamma \frac{\partial u_j}{\partial x} = 0, \quad (5) \\
\frac{\partial^3 \psi}{\partial x^3} = [1-\mu_n + \mu_d] \exp^\psi + \mu_n n_n - \mu_d n_d, \quad (6)
\]

where \( n_i \) is the number density with \( s = n(i)e(d) \) of negative ion (positive ion) electron (stationary dust), \( u_i \) is the fluid speed of \( s \), \( m_i \) is the positive (when \( j = i \)) or negative (when \( j = n \)) ion mass, \( Z_d \) is the number of electron occupy on the dust grain side, \( e = \) magnitude of the electron-charge (q), \( \phi \) is the electrostatic wave potential; \( n_{s0}, (n_d) \), and \( n_{d0} \) are the equilibrium value of \( n_s \), \( n_d \), and \( n_d \) respectively i.e. \( n_s \), \( n_d \), and \( n_d \) are the number density normalized by \( n_{s0}, (n_{d0}) \), and \( n_{d0} \) respectively, \( p_i \) is the pressure of species \( i \), \( \gamma \) is an adiabatic index, \( x \) is the space variable, and \( t \) is the time variable.

3. K-dV Equation

For the DIA K-dV equation we introduce the stretched coordinates:

\[
\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} - u_j \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial x} = 0, \quad (13)
\]

Where, \( V_p \) is the wave phase speed (\( \omega/k \)), and \( \epsilon \) is a smallness parameter (\( 0 < \epsilon < 1 \)). To get the dispersion relation, we expand \( n_s, u_s, p_s \), and \( \phi \) with \( s \) be the charged species like positive and negative ion, electron in power series of \( \epsilon \), to their equilibrium and perturbed parts.

\[
n_s = 1 + c n_s^{(1)} + \epsilon^2 n_s^{(2)} + \epsilon^3 n_s^{(3)} + \cdots, \quad (8) \\
u_s = 0 + c u_s^{(1)} + \epsilon^2 u_s^{(2)} + \epsilon^3 u_s^{(3)} + \cdots, \quad (9) \\
p_s = 0 + c p_s^{(1)} + \epsilon^2 p_s^{(2)} + \epsilon^3 p_s^{(3)} + \cdots, \quad (10) \\
\psi = 0 + c\psi^{(1)} + \epsilon^2 \psi^{(2)} + \epsilon^3 \psi^{(3)} + \cdots. \quad (11)
\]

Where \( n_s^{(1)} \), \( u_s^{(1)} \), \( p_s^{(1)} \), and \( \psi^{(1)} \) are the perturbed part of \( n_s, u_s, p_s \), and \( \psi \) respectively.

Combining above equations, we get -

\[
V_p = (-b \pm \sqrt{(b^2 - 4ac)/2a})^{1/2}, \quad (12)
\]

Where,

\[
\alpha = (1 - \mu_n + \mu_d), \quad a = \mu_n, \quad b = \mu_n + \mu - \alpha \gamma \delta_n - \alpha \mu \gamma \delta_n, \quad \text{and} \quad c = \alpha \gamma^2 \delta_n \delta_n - \gamma \delta_n \mu_n.
\]

Equation (12) represents linear dispersion relation.

The next higher order of \( \epsilon \) can be simplified as an equation of the form:

\[
\frac{\partial \psi}{\partial t} + A \psi \frac{\partial \psi}{\partial x} + B \frac{\partial^3 \psi}{\partial x^3} = 0, \quad (13)
\]
Where,
\[ A = \frac{Y}{X}, \]
\[ \beta = \frac{1}{X}, \]
\[ X = (2V_p/d_3^2) (2V_m \mu \mu_p/d_1), \]
\[ Y = -c_i/d_3 + \mu_c c_i/d_1 + \mu_c c_i/d_1, \]
\[ c_1 = V_p^4 + 2 V_p^2 \gamma \delta_n - 3 V_p^2 + \gamma^2 \delta_n - 2 \gamma \delta_n, \]
\[ c_2 = V_p^4 \mu^2 - 2 \mu V_p^2 \gamma \delta_n + 3 \mu V_p^2, \]
\[ c_3 = \gamma^2 \delta_n^2 + 2 \gamma \delta_n - \gamma^2 \delta_n. \]

Equation (13) is known as K-dV equation. We get stationary localized solution of (13) by introducing a transformation \( \xi = \zeta - U_0 t, \)
\[ \psi = \psi_m \text{sech}^2 \left[ \left( \zeta - U_0 t \right)/\delta \right], \quad (14) \]
Where the amplitude \( \psi_m \) and the width \( \delta \) are given by \( \psi_m = 3U_0/A, \) and \( \delta = \sqrt{4\beta}/U_0, \) respectively.

![Fig 1: The variant of the positive soliton with \( \mu \) when the adiabatic system.](image)

Equation (14) is the solution of K-dV equation. This represents a solitary wave. Here we see that the Fig 1 shows the variant of the positive K-dV solitons with mass number density (\( \mu \)) when the system contains positively charged dust in an adiabatic system.

4. mK-dV Equation

For the third order calculation a new set of stretched coordinates is applied:
\[ \zeta = \epsilon(x - V_p t), \quad \tau = \epsilon^3 t, \quad (15) \]

Using (15) we can find the same values of \( n_i^{(1)}, n_n^{(1)}, n_e^{(1)}, u_i^{(1)}, u_e^{(1)}, u_n^{(1)}, p_i^{(1)}, p_e^{(1)}, p_n^{(1)}, \) and \( V_p \) as like as that in K-dV.

To the next order approximation of \( \epsilon, \) we obtain a set of equations, which, after using the values \( n_i^{(1)}, n_n^{(1)}, n_e^{(1)}, \) and \( V_p, \) can be simplified and applying the condition, \( \psi \neq 0 \) (so, it’s coefficient is zero), we get,
\[ \frac{1}{2} \left\{ A(\psi^{(1)})^2 \right\} = 0 \quad (16) \]

For the next higher order of \( \epsilon, \) we obtain an equation:
\[ \partial \psi/\partial \tau + \alpha \beta \psi^2 \partial \psi/\partial \xi + \beta \partial^3 \psi/\partial \xi^3 = 0, \quad (17) \]

Where,
\[ \alpha = F(-a_1^2 + 15/2 - 21\gamma \delta_n/2a_1 - 5\gamma^2 \delta_n/2a_1 - 3\gamma \delta_n^2/a_1^2) + G(a_2^2 - 15/2 - 21\gamma \delta_n/2a_2 - G(5\gamma^2 \delta_n/2a_2 - 3\gamma \delta_n^2/a_2^2 - 3\gamma^2 \delta_n/a_2^2), \]
\[ \beta = V_p a_1^2 a_2^2 /(-2 \mu_m \mu_p V_p^2 - 2 \gamma \delta_n a_1^2). \]

Where,
\[ F = \mu_p a_1^3, \quad G = 1/a_2^3, \quad a_1 = (\gamma \delta_n - \mu V_p^2), \quad \text{and} \quad a_2 = (V_p^2 - \gamma \delta_n). \]

Equation (17) is known as mK-dV equation. The stationary localized solution of (17), obtained by introducing a transformation \( \xi = \zeta - U_0 \tau, \) is, therefore, directly given by
\[ \psi = \psi_m \text{sech} \left[ \xi/\Delta \right], \quad (18) \]

Where the amplitude \( \psi_m \) and the width \( \delta \) are given by \( \psi_m = \sqrt{6} U_0 / \alpha \beta, \) and \( \delta = 1/\psi_m \gamma, \) where the amplitude \( \psi_m \) and the width \( \Delta \) are given by \( \psi_m = \sqrt{6} U_0 / \alpha \beta, \) \( \Delta = 1/(\sqrt{\gamma} \psi_m), \) and \( \gamma = \alpha/6. \)

![Fig 2: The variant of the mK-dV soliton with \( \mu_d \) for adiabatic process.](image)
5. RESULTS AND DISCUSSION

Dust ion acoustic K-dV and mK-dV solitons have been investigated in a multi-ion dusty plasma system in adiabatic change that gives;

1. The positive and negative K-dV solitons are observed.
2. The width and amplitude of the K-dV solitons varies with polarity changes.
3. Amplitude of the positive and negative soliton increases with the increasing mass number density for positive dust but decreases for negative dust.
4. Existence of positive mK-dV solitons is observed.
5. The width of the positive mK-dV soliton decreases with adiabatic index for positive dust but both increases for negative dust.

6. CONCLUSION

Present investigation is valid for tiny amplitude DIA K-dV solitons. Though we have considered positive and negative ions, Maxwell’s electrons, and arbitrarily charged stationary dust, our model is applicable for small amplitude waves only, and the similar experimental setups of Barkan et al., (1996) or (Nakamura and Sharma, 2001) can be used to observe the solitons. Plasma with dust is violently known as dusty plasma (Bliokh et al., 1995; Verheest, 2002). A numerical approach allowing calculation of the grain charge while including peripheral electron emission and the major role of this approach to the lunar condition is provided. In conclusion, we propose that a new experiment may be performed based on our results to observe such waves and the effects of nonlinearity on these waves in both laboratory and space dusty plasma system.

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CONFLICTS OF INTEREST

The authors declare that they have no competing interests with respect to the research.

REFERENCES


